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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, AUGUST 2021

FIRST YEAR [BATCH 2020-23]

Date : 10/08/2021

PHYSICS (HONOURS)

Time : 11 am – 1 pm

PAPER : III [CC3]

Full Marks : 50

Answer **any five** of the following questions: [5×10]

1. a) Given a function $f(x) = x\sin x$

   Write down its Fourier series in the interval $0 < x < 2\pi$.

   b) Write down the Fourier series of $f(x) = 2x - x^2$ in the interval $0 < x < 3$. Hence show that

   $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots = \frac{\pi}{12}$ [5+5]

2. a) Consider the function $f(x) = \sin x, \quad 0 < x < \pi.$

   Sketch the even periodic extension of $f(x)$. Hence compute its Fourier series.

   b) Given $x = 2\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \sin nx$

   Compute the Fourier series of $f(x) = x^2$ in the interval $-\pi < x < \pi$. From the Fourier series expansion of $f(x)$ show that, $2\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{6}$. [5+5]

3. a) Evaluate the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & , -1 \le x \le 1 \\ 0 & , |x| > 1 \end{cases}$

   Now making use of the inverse Fourier transform evaluate the integral.

   $\int_0^{\infty} \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx.$

   b) Consider a metallic rod of length 2 m. The left and the right ends are maintained at a temperature of 0^0 C and 100^0 C, respectively. Determine the temperature $T(x,t)$ of the rod as a function of x and t. The initial conditions are $T(x,0) = \begin{cases} 100x , 0 < x < 1 \\ 100 \;, 1 < x < 2 \end{cases}$ [5+5]

4. a) Consider a string of length 2 m, whose two ends are fixed and the velocity is 30 m/s. The initial conditions are given as

   $u(x,0) = 0$

   $\frac{\partial}{\partial t} u(x,0) = 300\sin 4\pi x$

   Evaluate the solution of the wave equation of this vibrating string. Hence determine the maximum displacement of the string at a distance $x = \frac{1}{8}$ m.

b) Consider a vibrating string π m long, and both of its ends are fixed. If the initial displacement is $f(x)$ and its initial velocity is $g(x)$. Find the expression for $u(x,t)$. [5+5]

5. a) Use Frobenius method to solve the following differential equation for Hermite polynomial around x=0

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2ny = 0$$ where n is a non-negative integer.

b) Legendre polynomials may be expressed as $(1-2tx+t^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} P_n(x)t^n$

Use the above relation to show that i) $(n+1)\left\{P_n P'_{n+1} - P_{n+1}P'_n\right\} = P_0^2 + 3P_1^2 + 5P_2^2 + \ldots + (2n+1)P_n^2$ [6+4]

6. a) Show that Bessel functions defined by $e^{\frac{x}{2}(t-\frac{1}{t})} = \sum_{-\infty}^{\infty} J_n(x)t^n$ have the integral representation

$$J_n(x) = \frac{1}{2\pi}\int_0^{2\pi} \cos(x\sin\theta - n\theta)\, d\theta$$

b) If α and β are the roots of the equation $J_0(x) = 0$ show that

$$\int_0^1 xJ_0(\alpha x)J_0(\beta x)\,dx = \frac{1}{2}\delta_{\alpha\beta}J_1^2(\alpha)$$

c) Prove that $J_1'' = \left(\frac{2}{x^2} - 1\right)J_1(x) - \frac{1}{x}J_0(x)$ [2+5+3]

7. a) Express the following function in terms of Legendre polynomial $1+2x-3x^2+4x^3$

b) Evaluate (i) $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}}\,dx$

(ii) $\int_0^{\infty} e^{-x^4}\,dx$ [4+(3+3)]

8. a) In a bolt factory machines A, B and C manufacture 25%, 35% and 40% of the total. Of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and Is found to be defective. What are the probabilities that it was manufactured by machine A, B or C?

b) Define Poisson's distribution.

c) Calculate the mean and standard deviation of Poisson's distribution. [5+1+4]

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